Advanced Engineering Mathematics (v1. by 11/13/2002)

1. Find the solution of the following differential equation
   (a) $xy\, y' = 2y^2 + 4x^2$, $y(2) = 4$
   (b) $2x \tan y dx + \sec^2 y\, dy = 0$
   (c) $y' \cos x = y \sin x + \sin 2x$
   (d) $x^2 y' + xy = 1$, $x > 0$, $y(1) = 2$
   (e) $2xyy' + (x-1)\, y^2 = x^2 e^x$
   (f) $y''' - 6y'' + 12y' - 8y = x^{1/2} e^{2x}$
   (g) $y'' + 6y' + 9y = 50\, e^{-x} \cos x$
   (h) $x^2 y'' - 4xy' + 6y = 7x^4 \sin x$
   (i) $(x^3 D^3 - 3x^2 D^2 + 6xD - 6)y = 12/x$, $y(1) = 5$, $y'(1) = 13$, $y''(1) = 10$
   (j) $y_1' = -3y_1 - 4y_2 + 5e^t$, $y_2' = 5y_1 + 6y_2 - 6e^t$, $y_1(0) = 19$, $y_2(0) = -23$
   (k) $y_1' = -y_1 - 4y_2 + 2y_3$, $y_2' = 2y_1 + 5y_2 - y_3$, $y_3' = 2y_1 + 2y_2 + 2y_3$
   (l) $y_1' = 3y_1 + y_2 - 3 \sin 3t$, $y_2' = 7y_1 - 3y_2 + 9 \cos 3t - 16 \sin 3t$
   (m) $x (x-1) y'' + (3x - 1) y' + y = 0$
   (n) $x y'' + y' - xy = 0$
   (o) $x^2 y'' + x y' + (\lambda^2 x^2 - \nu^2)y = 0$
   (p) $4xy'' + 2y' + y = 0$
   (q) $x^2 y'' + 1/4(x + 3/4) y = 0$
   (r) $y'' + 2y' - 3y = 6e^{-2t}$, $y(0) = 2$, $y'(0) = -14$ (solved by Laplace transform only)
   (s) $y'' + 3y' + 2y = r(t)$, $r(t) = 1$ if $0 < t < 1$ and 0 if $t > 1$; $y(0) = 0$, $y'(0) = 1$
   (t) $y'' - 3y' + 2y = 4t$, $y(0) = 1$, $y'(0) = -1$

2. Find a general solution of (a) $y_1' = 2y_1 + y_2$, $y_2' = 6y_1 + 2y_2$, (b) $y_1' = -2y_1 + 2 y_2$, $y_2' = -2y_1 - 2y_2$. Determine the type and stability of the critical point.
3. Show that $J'_0(x) = -J_1(x)$, $J'_2(x) = 1/2[J_1(x) - J_3(x)]$
4. Evaluate $\int_1^2 x^{-4} J_5(x)\, dx$, where $J_1(2) = 0.5767$, $J_0(2) = 0.2239$, $J_1(1) = 0.4401$ and $J_0(1) = 0.7652$
5. Let $f = xy - yz$, $\mathbf{v} = [2y, 2z, 4x + z]$, $\mathbf{w} = [3z^2, 2x^2 - y^2, y^2]$. Find (a) grad f at (2,0,7), (b) $D_{\mathbf{v}}f$ at (2,3,1), (c) [(curl **v**) x **w**] • **w**, (d) div (grad ($x^2 f$))

6. Is **A** Hermitian or skew-Hermitian? Find $\bar{x}^T \mathbf{A x}$? (a) $\mathbf{A} = \begin{bmatrix} -i & 1 & 2+i \\ -1 & 0 & 3i \\ -2+i & 3i & i \end{bmatrix}$,

$\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, (b) $\mathbf{A} = \begin{bmatrix} 1 & i & 4 \\ -i & 3 & 0 \\ 4 & 0 & 2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 1 \\ i \\ -i \end{bmatrix}$

7. Integrate the surface integral $\iint_S F \bullet n dA$ (a) $\mathbf{F} = [e^y, 0, ze^x]$, S: $\mathbf{r} = [u, 2u, v]$, $-1 \leqq u \leqq 1$, $0 \leqq v \leqq 3$, (b) $\mathbf{F} = [y^2, x^2, z^2]$, S the surface of the cyclinder $x^2 + y^2 \leqq 4$, $0 \leqq z \leqq 2$, (c) $\mathbf{F} = [x^2, y^2, z^2]$, S: $\mathbf{r} = [u \cos v, u \sin v, 3v]$, $0 \leqq u \leqq 1$, $0 \leqq v \leqq 2\pi$
8. Find the complex Fourier series of (a) $f(x) = e^x$, (b) $f(x) = x^2$ if $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$ and obtain the real Fourier series from above expression.
9. Find the Fourier cosine and sine integrals of $f(x) = e^{-x} + e^{-2x}$ $(x > 0)$
10. Find the Fourier cosine transform of (a) $f(x) = x$ if $1 < x < a$ and $f(x) = 0$ otherwise. (b) $f(x) = \cos(ax^2)$ $(a > 0)$
11. Find the Fourier sine transform of (a) $f(x) = x + 1$ if $0 < x < 1$ and $f(x) = 0$ otherwise. (b) $f(x) = x \exp(-ax^2)$ $(a > 0)$
12. Find the Fourier transform of (a) $f(x) = \exp(-2x)$ if $x > 0$ and $f(x) = 0$ if $x < 0$. (b) $f(x) = \dfrac{\sin ax}{x}$ $(a > 0)$
13. Find the Fourier series of $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$
14. Find the Fourier transform of $f(x) = \exp(-ax^2)$ $(a > 0)$
15. Solve $\dfrac{\partial U}{\partial t} = c^2 \dfrac{\partial^2 U}{\partial x^2}$, $U_x(0,t) = 0$, $U_x(L,t) = 0$, $U(x,0) = x$.
16. Solve $\dfrac{\partial^2 U}{\partial x^2} = -\dfrac{\partial^2 U}{\partial y^2}$, $U(0,y) = 0$, $U(a,y) = 0$, $U(x,0) = 0$, $U(x,b) = x$.
17. Use Fourier transform to solve the following partial differential equations: $\dfrac{\partial U}{\partial t} = c^2 \dfrac{\partial^2 U}{\partial x^2}$, $U(0,t) = 0$, $U(L,t) = 0$, $U(x,0) = k$ if $|x| < 1$ and 0 if $|x| > 1$
18. Solve $\dfrac{\partial U}{\partial t} = \dfrac{\partial^2 U}{\partial^2 x}$, $U_x(0,t) = 0$, $U(x,0) = x$ if $0 \le x \le 1$ and 0 if $x > 1$, $U(x,t)$ is bounded where $x > 0$, $t > 0$.