Adaptive Integral Position Control Using RBF Neural Networks for Brushless DC Linear Motor Drive

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Abstract—The paper presents an adaptive integral position controller using RBF (Radial Basis Function) neural networks (NNs) for a brushless DC linear motor. By assuming that the upper bounds of the nonlinear friction and force ripple, an RBF NN is used for approximating the friction, the force ripple and the load; an adaptive backstepping control with integral action is then proposed to achieve position tracking of the linear motor. The parameter adjustment rules for the overall controller are derived via the Lyapunov stability theory. Based on the LaSalle-Yoshizawa lemma, the proposed controller is proven asymptotically stable. Experimental results are conducted to show the efficacy and usefulness of the proposed control method.


I. INTRODUCTION

Over the past decades, linear motors have been widely used for high-speed and high-accuracy applications, such as precision $X-Y$ tables, fast manipulators, semiconductor manufacturing equipment, computer numerical control (CNC) machine tools, and so on. Brushless DC linear motor is one of the most useful linear motors for industrial automation and precise motion drive. Generally speaking, brushless DC linear motor has the advantages of silence, simple mechanism, no dust, and high-speed and accurate motion ability. In addition, such a linear motor has several unsolved technical difficulties, as compared to conventionally rotary DC servomotors, in its nonlinear and time-varying characteristics, such as nonlinear friction, ripple force generated by magnetic pole and end-effects of the motor.

Precise motion control for brushless DC linear motors has attracted much attention in both academia and industry. Friction compensation is one of the key issues on high precision motion control of such motors. The well-known LuGre friction model (Z-model) was proposed by Canudas et al. [1], which could describe many friction’s behaviors and was useful for various control tasks.

Adaptive tracking control with neural networks for the motor with the friction has been investigated by several researchers. Based on the LuGre model, Caundas and Ge [2] proposed an observer-based adaptive friction compensation controller to deal with system’s position / velocity dependency characteristics; they used a multi-layer Gaussian RBF neural network to approximate the unknown function $\alpha(x, \dot{x})$ of the Z-model without any priori knowledge on the friction, thus successfully improving the Z-model by adding velocity / position friction dependently. Ge et al. [3] and Omatu et al. [4] proposed an investigation survey for adaptive friction compensation; their adaptive controllers were proposed and applied in both model-based friction and neural network friction (non-model-based) system. In the paper, the Gaussian RBF neural networks of 100 nodes were chosen to approximate the friction. The controller aimed at compensating for friction directly, and the simulation result showed the advantage of the RBF neural network. Ge et al. [5] presented an adaptive RBF NN controller. Being different from [1], the control law in [5] incorporated the RBF NN and the weight vector was updated adaptively. Since that time, the RBF NN had played an important role in control design, not only appeared in friction approximation. In 2002, Tan et al. [6] developed a robust adaptive compensation for friction force and force ripple in permanent-magnet linear motors. The force ripple was estimated and compensated. Sato et al. [7] proposed an adaptive $H_\infty$ control for linear motor in 2004, in which the effect of the friction was expected to be compensated and the RBFNN was used to approximate the friction. Furthermore, the parameter updating rules in [7] were improved by the $\sigma$ modification strategy to ensure the boundness of the estimated parameters. Last but not least, Tsai et al. [8] in 2005 proposed an adaptive nonlinear $H_\infty$ control for a brushless DC linear motor in order to achieve velocity tracking control with the assumption of known upper bounds of the ripple force and nonlinear friction. However, the control effort in [8] is very large in order to deal with the unknown but bounded uncertainties.

The paper is written in two principal contributions. The first one is that the well-known RBF NN is employed in controller design to approximate the friction and the force ripple in order to circumvent unreasonable control effort occurred in [8] and [9]. The second one is that the adaptive integral control is then proposed to accomplish position trajectory tracking. The main feature of the proposed method hinges on the facts that the adaptive laws for on-line updating the RBF NN and the tracking controller are derived via the Lyapunov stability theory. Worthy of mention is that the adaptive position controller inherently possesses tracking...
can denote the motor driving force; \( f \) for ripple, the reader is referred to [7],[9] and as the position and velocity of motor, respectively, \( u \) denotes the positive roundedness of the force ripple; the \( u \) force from the linear guide. For detailed descriptions of these two forces \( f_{\text{fric}} \) and \( f_{\text{ripp}} \), the reader is referred to [7],[9] and therein. Furthermore, it is well-known that \(|f_{\text{fric}}| \leq k_0 + k_1 |x|\) and \(|f_{\text{ripp}}| \leq |A_r|\) where \( k_0 \) and \( k_1 \) are two positive constants, \( A_r \) denotes the positive roundedness of the force ripple; the external load \( f_{\text{load}} \) is reasonably assumed to be bounded.

With these symbols, one defines the state variables \( x_1 \) and \( x_2 \) as the position and velocity of motor, respectively, and then rewrites the dynamic equations of the motor model (1) as below;

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = (k_f i - f_{\text{fric}} - f_{\text{ripp}} - f_{\text{load}})/m = (u - f_{\text{fric}} + f_{\text{ripp}} + f_{\text{load}})/(k_f k_u) = (u - \overline{T})/b
\]

where

\[
\overline{T} = (f_{\text{fric}} + f_{\text{ripp}} + f_{\text{load}})/k_f k_u \\
b = m/(k_f k_u) \quad \overline{T} \leq \overline{T}_{\text{max}} + k |x_t| \]

According to the results from Canudas et al. [5], \( \overline{T}_{\text{max}} \) can be
well approximated by using the RBF NN, i.e.,
\[
\tilde{f}_{max} = W^T S + \varepsilon = [w_1 \cdots w_n][s_1 \cdots s_n]^T + \varepsilon
\]
where
\[
s_i = \exp\left[-\left((x-u_{i1})^2 + (\dot{x}-u_{i2})^2\right)/\sigma_i^2\right]
\]
\(W\) denotes the weighting vector of the RBF NN, \(S\) represents the radial basis function, and \(\sigma_i\) means the standard variation of the \(i\)th node of RBF NN. Furthermore, the approximation error \(\varepsilon\) is assumed to have an upper bound, i.e.,
\[
|\varepsilon| \leq \varepsilon_d
\]
Notice that the dimension of RBF NN significantly affects the accuracy of friction approximation. The parameters \(\sigma_i, u_{i1}\) and \(u_{i2}\), \(i=1,\ldots,n\), can be determined in advance by the off-line learning process. The closer NN approximation to the actual friction, the smaller \(\varepsilon_d\) can be expected.

III. ADAPTIVE INTEGRAL POSITION CONTROL VIA BACKSTEPPING

For the brushless DC linear motor model (2), the position control is much more complex than the velocity tracking in practice due to the nonlinear friction, the ripple force and even unpredictable load changes. This motivates us to apply the backstepping method to develop an adaptive position control method with integral action; the integral action is specially introduced to eliminate the steady state errors in step commands. To clarify the procedure of the method, we develop the adaptive integral controller step by step. At first we utilize a simple second-order system to illustrate the stability by introducing an integral action into system. Second, we define a backstepping error in order to incorporate with the integral action in control design and prove the stability of the proposed controller via the Lyapunov stability theory. At the last step, the controller is further modified to an adaptive form in which its adaptation rules are presented. The following steps elucidate the design procedures of this control method in detail.

**Step1:** To achieve position control for a time-differentiable given position trajectory \(x_{ref}\), one defines the position tracking error \(x_{le}\) as \(x_{le} = x_1 - x_{ref}\) and then obtains the time derivative of the error as follows;
\[
\dot{x}_{le} = \dot{x}_1 - \dot{x}_{ref} = x_2 - \dot{x}_{ref}
\]
Next, consider \(x_2\) as the virtual control such that the state variable \(x_{le}\) is asymptotically stable. Thus, by designing
\[
x_2 = -k_p x_{le} - k_f \int x_{le}(\tau)d\tau
\]
which leads to the sequel second-order characteristic equation
\[
x_{le} + k_p \dot{x}_{le} + k_f x_{le} = 0
\]
It is obvious that if \(k_p > 0\) and \(k_f > 0\), then the two poles of the equation have negative real parts, i.e., the two poles lie within the left-half side of S plane.

In what follows describes an alternative to show that equation (8) is asymptotically stable by choosing the following Lyapunov function candidate
\[
V = \frac{1}{2} x_{le}^2 + \frac{1}{2} k_f \left[ \int x_{le}(\tau)d\tau \right]^2
\]
The time derivative of \(V\) along its motion trajectory is given by
\[
\dot{V} = -k_p x_{le} + k_f \left[ \int x_{le}(\tau)d\tau \right] \dot{x}_{le} + k_f \left[ \int x_{le}(\tau)d\tau \right] \dot{x}_{le} + k_f x_{le} \left[ \int x_{le}(\tau)d\tau \right] \dot{x}_{le}
\]
Since \(\dot{V}\) is negative semidefinite, it is concluded by LaSalle-Yoshizawa lemma that \(x_{le} \to 0\) as \(t \to \infty\).

**Remark1:** The parameters \(k_p\) and \(k_f\) can be designed such that equation (10) has a desired characteristic function.

**Step2:**
This step aims at using the backstepping approach to find a controller suitable for an adaptive strategy. The following backstepping error \(\xi = x_2 - (-k_p x_{le} - k_f \int x_{le}(\tau)d\tau + \dot{x}_{ref})\) leads to attain the controller design. Differentiating the backstepping error with respect to time gives
\[
\dot{\xi} = \dot{x}_2 - (-k_p \dot{x}_{le} + k_f \dot{x}_{le} + \dot{x}_{ref}) = [u - f] + b(k_p \dot{x}_{le} + k_f x_{le} - \dot{x}_{ref})/b
\]
The control \(u\) is designed in the following form
\[
u = -b(k_p \dot{x}_{le} + k_f x_{le} - \dot{x}_{ref}) - W^T S \text{sgn}(\zeta)
\]
\[
- k_\zeta \dot{\zeta} - x_{le} - e_d \text{sgn}(\zeta) - k_\zeta \|\text{sgn}(\zeta) - \zeta\|/b
\]
such that (13) becomes
\[
\dot{\xi} = [- W^T S \text{sgn}(\zeta) - k_\zeta \xi - e_d \text{sgn}(\zeta) - k_\zeta \|\text{sgn}(\zeta) - \zeta\|/b
\]
In order to prove the asymptotic stability of the overall system, we propose the following Lyapunov function candidate
\[
V = \frac{1}{2} x_{le}^2 + \frac{1}{2} k_f \left[ \int x_{le}(\tau)d\tau \right]^2 + b/2 \dot{\zeta}^2
\]
Taking the time derivative of \(V\) along its trajectory gives
\[
\dot{V} = -k_p x_{le}^2 - W^T S [\xi - k_\zeta \xi - e_d \|\text{sgn}(\zeta) - k_\zeta \|\text{sgn}(\zeta) - \zeta]^{-T}
\]
\[
\leq -k_p x_{le}^2 - W^T S [k_\zeta \xi - e_d \|\text{sgn}(\zeta) - k_\zeta \|\text{sgn}(\zeta) - \zeta]^{-T}
\]
\[
= -k_p x_{le}^2 - (e_d - \xi) \leq 0
\]
which implies that \(\dot{V}\) is negative semidefinite. Hence, the use of Barbalat’s lemma indicates that \(x_{le} \to 0\) and \(\zeta \to 0\) as \(t \to \infty\).

**Step3:**
This step is devoted to finding a set of parameters adaptation rules such that \(\zeta \to 0\) and \(x_{le} \to 0\) as \(t \to \infty\). Thus,
the adaptive control law is proposed as follows;

\[ u = -\dot{b}(k_p\dot{x}_{le} + k_I x_{le} - \dot{x}_{1ref}) - \dot{\hat{W}}^T S \text{sgn}(\xi) - k_\xi \dot{\xi} - x_{le} - \dot{\xi} \text{sgn}(\xi) - \dot{\hat{k}}|\dot{x}_1| \text{sgn}(\xi) \]

where the corresponding updating rules are given by:

\[ \dot{\hat{b}} = r_\xi (k_p\dot{x}_{le} - k_I x_{le} - \dot{x}_{1ref}) \]
\[ \dot{\hat{W}} = r_s S \dot{\xi} \]
\[ \dot{\hat{\xi}} = r_\xi |\dot{\xi}| \]
\[ \dot{\hat{k}} = r_k |\dot{\xi}| \]

where \( \dot{\hat{b}}, \dot{\hat{W}}, \dot{\hat{\xi}}, \) and \( \dot{\hat{k}} \) are the estimates of the parameters \( b, W \) and \( \xi \); \( k \) is the factor governing how soon the position error converges to zero. The auxiliary parameters \( r_\xi, r_s, \) and \( r_k \) are all positive constants which govern the parameter updating rates, and they can be selected by the designer. The parameter updating rules (19) are chosen to achieve the asymptotical stability. Because of the ability of adaptive estimates, the control is expected to work well under external uncertainties. The following theorem summarizes the result that the position errors will asymptotically decay to zero.

**Theorem 1:** Assume that the parameters \( k_p \) and \( k_I \) are positive gains. Then all the trajectories of the closed-loop error system composed of the system (2) and the adaptive control (18) with the parameter adaptation rules (19), are globally uniformly bounded, thereby implying that all the estimates, \( \dot{x}_{le}, \dot{\hat{b}}, \dot{\hat{W}}, \dot{\hat{\xi}}, \) and \( \dot{\hat{k}} \), are uniformly bounded for \( t \geq 0 \). Furthermore, \( x_{le} \to 0 \), and \( \xi \to 0 \) as \( t \to \infty \).

**Proof:**

To show the asymptotical stability of the closed-loop error system, we find a radially, unbounded Lyapunov function and use it to derive a set of parameter adaptation laws. Hence we have the Lyapunov function

\[ V_e = \frac{1}{2} x_{le}^2 + \frac{1}{2} k_I (x_{le} - x_{1ref})^2 + \frac{1}{2} (\dot{\xi}^2 + \dot{\hat{b}}^2) + \frac{1}{2} (\dot{\hat{W}}^T \dot{\hat{W}} + (W^T W + 2r_s)) + (\dot{\hat{\xi}}^2 + \dot{\hat{k}}^2) \]

where \( r_\xi > 0, r_s > 0, r_k > 0, \) \( \dot{\hat{b}} \geq 0 \), \( \dot{\hat{W}} = W - \hat{W} \), \( \dot{\hat{\xi}} = k - \hat{k} \) and \( \dot{\hat{b}} = \dot{\hat{b}} - \hat{b} \), \( \dot{\hat{\xi}} = \dot{\hat{\xi}} - \hat{\xi} \), \( \dot{\hat{k}} = \dot{\hat{k}} - \hat{k} \), \( \dot{\hat{W}} = \dot{\hat{W}} - \hat{W} \). Taking the time derivative of \( V_e \) and using

\[ \dot{x}_{le} = x_2 - x_{1ref} = \xi - k_p x_{le} - k_I x_1 - \dot{x}_{1ref} \int x_{le}(t) dt \]

we obtain

\[ V_e = -k_p x_{le}^2 - k_I \xi^2 + \dot{\hat{b}}(k_p x_{le} + k_I x_{le} - \dot{x}_{1ref} - \dot{\hat{b}}(r_\xi)) - |W^T S + \epsilon + k | \dot{\xi}| | - |T| \]
\[ + \dot{\hat{\xi}}(\dot{\xi} + \dot{\hat{\xi}}) + \dot{\hat{k}}(\dot{\xi} + \dot{\hat{k}}) \]

Hence, if the parameter adaptation rules (19) are used, then

\[ V_e = -k_p x_{le}^2 - k_I \xi^2 - |W^T S + \epsilon + k | \dot{\xi}| | - |T| \]
\[ \leq -k_p x_{le}^2 - k_I \xi^2 \leq 0 \]

which is negative semidefinite. Barbalat’s lemma implies that all the signals, \( \dot{x}_{le}, \dot{\hat{W}}, \dot{\hat{\xi}} \), and \( \dot{\hat{k}} \) are uniformly bounded and \( x_{le} \to 0 \) and \( \xi \to 0 \) for \( t \to \infty \). This completes the proof.

**Remark 2:** The estimates \( \dot{\hat{b}}, \dot{\hat{W}}, \dot{\hat{\xi}}, \) and \( \dot{\hat{k}} \) usually do not converge to their true values, unless that the desired position trajectory meets the persistent excitation condition.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

In this section a series of experiments are performed to examine the efficacy and usefulness of proposed position controller. Because of the limit of the linear motor thrust force, we have to assign some light loading experiments. Undoubtedly, these experiments have shown the efficacy of the controller. The following subsections describe these experiments in more detail and the following figures assist us to examine and evaluate the position tracking performance.

A. Experiments for Trapezoidal Position Tracking

Physically, the position trajectory has to be continuous. With this nature, it is not allowable to assign a “step” reference position trajectory. Instead, a trapezoidal reference position trajectory serves as a reference signal for the system. The two experiments in this subsection are to track the trapezoidal position trajectories in order to illustrate the effectiveness of the proposed controller. The parameters of the linear DC motor are given by \( L = 2.8 \text{mH}, \quad R = 3.33 \Omega, \quad K_e K_f = 0.1, \quad m = 4 \text{Kg}, \quad A_p = 1 \text{N}, \quad w = 176 \text{rad/mm} \).

The initial parameters of the adaptive position controller are listed in Table 1. To generate the trapezoidal position trajectory, an input command with a stroke of 0.16m and a period of 2 seconds is given by

\[ X(t)_{\text{command}} = \begin{cases} 
0.16t / 0.5 & 0 \leq t < 0.5 \\
0.16 & 0.5 \leq t < 1.5 \\
0.16(1.5-t) / 0.5 & 1.5 \leq t < 2 \\
0 & 2 \leq t \leq 2.16 
\end{cases} \]

which completes one cycle. The position command is fed into the second-order reference model \( G(s) = 10000 / (s^2 + 195s + 10000) \) to generate desired position trajectory. The first experiment was conducted to examine the tracking performance of the proposed controller for the trapezoidal trajectory. Figure 3 displays the experimental result of the linear motor position measurement. Figure 4 shows the position tracking error. It is clear that the controller is able to track the desired position trajectory in an acceptable accuracy.

The second experiment was conducted to verify the robustness of the proposed controller by changing the motor’s mass from 4Kg to 5.7Kg. Figure 5 illustrates the corresponding position tracking error. This figure shows that under the condition that the external load was changed, the control system follows the reference trajectory well.

B. Experiments for Sinusoidal Position Tracking

In the following two experiments, the position command is described by \( X(t)_{\text{command}} = 0.125 \sin(2\pi t + 1.5\pi) \). The phase
Fig. 3. Experimental position tracking response of Experiment 1.

Fig. 4. Experimental position tracking errors of Experiment 1.

Fig. 5. Experimental position tracking error with mass change from 4Kg to 5.7Kg in Experiment 1.

TABLE 2. INITIAL PARAMETERS OF THE ADAPTIVE CONTROLLER FOR EXPERIMENT 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$m$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$k_f$</td>
<td>3600</td>
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<tr>
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<td>10</td>
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<tr>
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</tr>
<tr>
<td>$\mu_2$</td>
<td>0</td>
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<tr>
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<td>0</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Fig. 6. Experimental position tracking response in Experiment 2.

Fig. 7. Experimental position tracking errors in Experiment 2.

Fig. 8. Experimental position tracking error with load increased from 4Kg to 5.7Kg in Experiment 2.

Fig. 9. Experimental behavior of the estimate $\hat{b}$ with the load increased from 4Kg to 5.7Kg in Experimental 2.
shift of $1.5\pi$ aims at assigning a smooth start for the motion control. The initial parameters of the controller are listed in Table 2. The first experiment was carried out in order to verify the tracking performance of the proposed controller for the sinusoidal trajectory. Figure 6 shows the position tracking responses. Figure 7 depicts the experimental results of the tracking errors for the time interval from zero and four seconds. The tracking errors were found to lie within $-20\mu m \sim +20\mu m$, but the accuracy was achieved at the cost of exaggerative control output fluctuation. To verify the robustness of the proposed controller under the sinusoidal command, the second experiment was carried out with the motor’s mass change from 4Kg to 5.7Kg. Figures 8 and 9 present the experimental position tracking error and the behavior of the estimate $\hat{b}$, respectively.

V. CONCLUSIONS

This paper has developed an adaptive backstepping RBF NN position control law with integral action for the linear DC motor. Several experimental results have shown the efficacy of the proposed control method. Since an integral action is introduced into the backstepping error $\xi$, this kind of action intends to eliminate the steady state error in position tracking. However, this steady-state error may be not acceptable in certain applications if the error is not small enough. It is theoretically predicted that the steady state error will approach zero if time is long enough. This can be interpreted as the integration term “saves” a large value whenever the position tracking error become large, it needs time to eliminate the effect of error integration. On the contrary, the integral action makes the backstepping error $\xi$ positive or negative for a longer period of time which greatly reduces the sign fluctuation of the control output. From the practical viewpoint, the integral action indeed benefits the system by reducing control output fluctuation. The controller may be useful for some applications.

The major weakness of the controller is that its adaptation law has a function of absolute value of the backstepping error $\xi$, which keeps monotonically increasing the values of some parameters. On such a condition, these parameters will not converge to their true values as time increases. A possible way to avoid the shortcoming is to assign appropriate values of the parameters $k_P$ and $k_I$ to improve the tracking performance. Moreover, the initial setting for the estimates $\hat{b}$ and $\dot{\hat{b}}$ is important. It is better to assign the parameters $r_a$ and $r_b$ small values so that $\hat{b}$ and $\dot{\hat{b}}$ will not increase too fast. Finally, worthy of mention is that even under the condition that the estimates may not converge to their true values, the controlled motor always tracks the desired position trajectories.

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