Assessment on the Instability of a Suspension Bridge with a Hexagonal Cross-section under Wind Action

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ABSTRACT: A numerical method is developed to predict the dynamic responses of a suspension bridge under wind action in a two-dimensional sense. In particular, the behaviour of a vibrating deck with a hexagonal cross-section is examined. Moreover, the influence of the attack angle, mass eccentricity and the vertical-torsional frequency ratio are discussed. Results show that the case with the largest attack angle (8°) leads to the greatest fluctuating responses. A similar tendency is also detected when the mass center is shifted downstream. Finally, as the frequency ratio changes from 1.2 to unity, the critical velocity decreases about 14%.

1 INTRODUCTION

The instability of a suspension bridge under wind action is an important subject in wind engineering. In order to investigate the dynamic motion of the bridge deck as well as the corresponding wind effects, the method of model experiments is commonly used. Economically, however, the application of appropriate numerical methods can be another option. In addition, the numerical results can provide more extensive information for the analysis of such flow-structure interactive problems.

A number of researchers have investigated experimentally the mechanisms of problems related to bridge instability. Typically, Scanlan and Tomko (1971) proposed a semi-experimental and semi-analytical approach regarding flutter derivatives, and this approach is presently widely used. Sarkar et al. (1992) suggested a system identification procedure to estimate all the flutter derivatives simultaneously. In their study, numerical simulations and reduction of the experimentally obtained direct derivatives are presented. Based on the experimental results from a coupled vertical-torsional free vibration of a spring-suspended section model, Gu et al. (2000) employed a least-square theory and proposed an identification method to obtain flutter derivatives. Phongkumsinga et al. (2001) proposed a method of suppression of flutter in long-span bridges. As an auxiliary mass is placed on the windward side of a bridge deck to shift the center of gravity, the aerodynamic moment acting on the deck is reduced, resulting in an increase in the flutter wind speed.

Besides wind tunnel tests, the application of computational fluid dynamics (CFD) is another way to assess the aerodynamic performance and aeroelastic stability of long-span bridges. Kuroda (1997) presented a numerical simulation of high-Reynolds-number flows past a fixed section model with a shallow hexagonal cross-section. The overall characteristics of measured static force coefficients were well captured by the computations. Lee et al. (1997) investigated numerically the wind load characteristics for turbulent flows over a two-dimensional bridge deck cross-section. The two-dimensional flow results were further used to perform three-dimensional dynamic structural analyses on a non-interactive basis. Fang et al. (2005) presented a numerical method to simulate the surrounding flow and to predict the corresponding dynamic responses of a bridge deck with a trapezoidal cross-section. By employing an interactive procedure, the dynamic responses of the deck agreed well with the measurement results.

In the present study, a numerical model is proposed to simulate the dynamic response of a suspended bridge in a two-dimensional sense. Particularly, a bridge deck with a hexagonal cross-section with mass eccentricity is investigated. In the numerical computations, two sets of equations, one for the simulation of the unsteady surrounding turbulent flow and the other for the calculation of the vibrating motions of the bridge deck, are solved alternatively to reflect the interaction effect between the structure and the flow by a completely interactive procedure. The resulting time-series responses of the structure as well as the wind loads are analyzed to examine the dynamic behaviors of the two. To verify the accuracy of the numerical results, on the other hand, wind tunnel experiments

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are conducted on a sectional deck model and the results of the vertical and torsional deflections of the deck are used to confirm the numerical predictions.

2 PROBLEM DESCRIPTION

Figure 1 depicts the schematic of the problem. The width of the hexagonal (B) is set as eight times of the deck thickness (D). The angle associated with the side surfaces of the hexagonal deck cross-section (θ) is selected typically as 120°. Two cases of mass eccentricity (e/B=0.1 and −0.1; a negative value denotes that the mass center is shift to the leeward side of the deck cross-section) are chosen to examine the eccentric effect. With five attack angles (β=0°, ±4°, ±8°), the approaching flow is considered smooth and the speed (U) varies from 2 to about 34 m/s (2 to about 18 m/s for model experiments). Other related properties of the bridge deck are described in Table 1.

3 NUMERICAL METHOD

The simulations contain two parts of computations. To predict the unsteady turbulent flow around the deck, a weakly-compressible-flow method (Song & Yuan, 1988) together with a dynamic subgrid-scale turbulence model (Germano et al., 1991) is used. After an instantaneous flow field is simulated, the resulting pressure distribution on the deck surfaces is integrated to obtain the corresponding wind load. The instantaneous load is then taken as an input to compute the corresponding structure responses. The resulting deflection and the vibrating speed of the deck are further fed back to the boundary specifications of the deck surfaces for the flow calculations in the following time step. Accordingly, the alternative solutions of the instantaneous flow field and the deck motions are considered the time-series results to describe the interactive dynamic behaviours of the two.

In the flow calculations, the continuity and momentum equations are (Song & Yuan, 1988)

\[ \frac{\partial p}{\partial t} + k \nabla \cdot \vec{V} = 0 \]  
\[ \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla \frac{p}{\rho} + \nabla \cdot \left[ (\nu + \nu_t) \nabla \vec{V} \right] \]  

where \( p \), \( \vec{V} \) and \( t \) denote respectively pressure, velocity and time; \( k \) is the bulk modulus of elasticity of air. \( \nu \) and \( \nu_t \) are respectively the laminar and turbulent viscosities, and the latter is determined based on the concept of a dynamic subgrid-scale turbulence model (Germano et al. 1991).

The flow computation proceeds according to a finite-volume approach with an explicit finite-difference scheme. To ensure numerical stability, the time increment in the unsteady calculations is limited by the Courant-Friedrich-Lewy criterion (1967).

In the flow calculations, appropriate values of pressures and velocities are specified at exterior (phantom) grids outside the boundaries of the computational domain to reflect the physical nature of the boundaries. For the velocity specifications, as the computation is based on a non-stationary coordinate system, the boundary specifications at the deck surfaces have to account for the effect due to the instantaneous motion of the bridge deck. At the upstream inlet boundary, a smooth and uniform velocity profile together with an additional velocity caused by the coordinate transformations is imposed. At the other penetrable boundaries (side and exit boundaries of the flow domain), zero-gradient boundary specifications with a similar treatment due to the coordinate transformations are adopted. For the pressure specifications, on the other hand, the average pressure at the downstream section of the flow domain is chosen as the reference pressure. At the other penetrable and solid boundaries, the values at the phantom cells are given according to a zero-gradient assumption in the direction normal to the boundaries.

In the computations of deck motions, on the other hand, the dynamic equations in the vertical (across-wind) and torsional directions are respectively

\[ e\ddot{\alpha} + \gamma_v + 2\xi_v \omega_v \dot{\gamma}_v + \omega_v^2 \gamma_v = \frac{F_L}{M} \]  
\[ \frac{e}{r_g^2} \ddot{\gamma}_v + \ddot{\alpha} + 2\xi_t \omega_t \dot{\alpha} + \omega_t^2 \alpha = \frac{F_M}{I} \]

where \( \gamma_v \) is the mean deflection of the deck centroid; \( \alpha \) is the torsional deflection; \( \dot{\gamma}_v \), \( \ddot{\gamma}_v \), \( \dot{\alpha} \), and \( \ddot{\alpha} \) are respectively the corresponding speeds and accelerations. \( F_L \) and \( F_M \) are the calculated wind loads in the across-wind and torsional directions; \( \omega \) is the circular frequencies; \( r_g \) is the radius of gyration of the deck about the center of rotation. The subscripts “v” and “t” denote the vertical and torsional directions.
4 EXPERIMENTAL PROGRAM

A cross-sectional deck model is installed on a suspended rack mechanism (see Fig. 2) in the test section (80 cm × 80 cm) of a wind tunnel. The turbulence intensity of the approaching flows in the tests is less than 0.5%. The thickness (D) and width (B) of the deck model are respectively 6 cm and 48 cm. The blockage in the tests is below 4%. Other related physical quantities are illustrated in Table 1. An additional energy absorber, filled with a viscous liquid, is set to reflect appropriate damping in the vertical and torsional directions. Hot-film anemometry is used to measure the approaching flow speed. Four laser transducers, on the other hand, are set on the rack to monitor the motions of the vibrating deck model.

5 RESULTS

5.1 Deck motion

Figures 3-5 show the comparisons of the root-mean-square values of the vertical and torsional deck deflections at various approaching wind speeds as the frequency ratio \( \frac{f_t}{f_v} \) is equal to 1.21. It is noted that the figures are presented in terms of the reduced velocities associated with the fundamental frequency in the across-wind direction \( (U_r = U/(f_t B)) \). It can be seen that a good agreement between the results of measurements and numerical predictions is obtained in all cases. Generally, the root-mean-square deflections in both directions increase as the approaching speed increases, except when two resonances occur \( (U_r = 1.26 \text{ and } 1.57) \) and lead to the presence of two local peak values. To avoid damage of the deck model, unfortunately, the wind speed in the experiments is limited to about 18 m/s, corresponding to a reduced velocity of 4.37. The numerical results, on the other hand, show that the root-mean-square deflections increase dramatically as the reduced velocity reaches about 5.35. When \( U_r \) exceeds about 6.50, numerical predictions show that the fluctuating responses in both the directions diverge, indicating the occurrence of flutter.

Evidence in Figures 3-5 also show that in the case when the attack angle equals to \( 8^\circ \) (or \(-8^\circ \)), it produces the largest deflected responses; the second largest deflections are found as \( \beta \) is equal to \( 4^\circ \) (or \(-4^\circ \)), and the smallest responses occur at an attack angle of \( 0^\circ \).

On the effect of eccentricity, Figures 3-5 reveal that the responses corresponding to an eccentricity in the leeward case (\( e/B=-0.1 \)) are the greatest; those corresponding to the windward case (\( e/B=0.1 \)) are the least, and the ones without eccentricity are in the middle.

Figures 6-8 show the other set of responses corresponding to the case that the frequency ratio \( \frac{f_t}{f_v} \) is equal to 1.01. Besides good agreements between the numerical and experimental results, they also show that the pattern of the variations of the fluctuating responses are similar to those in the previous case \( \left( \frac{f_t}{f_v} = 1.21 \right) \), except that there is only one peak value. Also, the effect of the attack angle and the eccentricity are the same. In overall comparison, however, the responses in the case when \( \frac{f_t}{f_v} = 1.01 \) are greater than those in the case as \( \frac{f_t}{f_v} = 1.21 \).

5.2 Flutter Derivatives

Figure 9 shows the variations of flutter derivatives typically as \( \beta \) is equal to \( 8^\circ \) with a leeward eccentricity and \( \frac{f_t}{f_v} = 1.01 \) (the most critical case). It can be seen that those related to the aeroelastic force in the across-wind direction \( (H_1, H_2) \) are all negative (Fig. 9a). In the torsional direction (Fig. 9b), on the other hand, all derivatives are positive except the one associated with the torsional speed \( (A_2) \), which is initially negative at lower wind speeds then becomes positive when \( U_r \) exceeds about 4.7.

5.3 Aerodynamic Damping

Based on the numerical results, Figure 10 shows typically the variations of the net damping ratios at various reduced velocities in the three eccentricity cases \( \left( \frac{f_t}{f_v} = 1.01, \beta = 8^\circ \right) \). In the across-wind direction (Fig. 10a), the general variation of the net damping ratio starts from the value of the material damping (0.6%) then increases with an increase of the wind speed. In the torsional direction (Fig. 10b), in contrast, the variation pattern of the net damping ratios appears rather different. The peak value is detected as it exceeds the resonance wind speeds. When the reduced velocity is about 4.7, the net damping ratios in all the three eccentricity cases become less than the material damping (0.5%). Finally, they drop to zero values as \( U_r \) approaches to about 5.65.

5.4 Critical velocity

According to the numerical results, Table 2 exhibits the normalized critical speeds in all the test cases. It can be seen that the case with an attack angle of \( 8^\circ \), a frequency ratio of 1.01 and a leeward eccentricity \( (e/B=-0.1) \) results in the least normalized critical velocity. On the other hand, the case corresponding to \( \beta=0^\circ \), \( e/B=0.1 \) (windward eccentricity) and \( \frac{f_t}{f_v} = 1.21 \) lead to the largest normalized critical
speed. In the table, the values in the brackets also reveal that the reduction in terms of the critical speeds as the frequency ratio changes from 1.01 to 1.21 is at an order of 14%.

6 DISCUSSION

As the results from the model measurements are treated as the prototype data for verifications, the application of the proposed numerical method has been proved to be successful in predicting the dynamic responses of the deck and the surrounding flow, which are actually interacted with each other. The variations of the root-mean-square deflections are well predicted (Figs 3-8), even when resonances occur. Also, other important features associated with the deck motion, such as the variations of flutter derivatives and net damping ratios, are obtained (Figs 9-10) to supplement the data of analyses.

Examinations on the general tendency of the variations of root-mean-square deflections (Figs 3-8) reveal that the fluctuating responses in both directions increase generally with an increase of the approaching speed. As the wind velocity approaches the resonance speed(s), the peak response(s) is(are) obtained obviously due to the effect of vortex-induced vibration.

Figures 3-8 also show the effect of the attack angle. As $\beta$ increases from zero to $^8$ (or decreases from zero to $-^8$), it results in an increase of the fluctuating responses and leads to smaller critical speeds (see also Table 2). Moreover, based on the results from the same figures, as the mass center shifts to the leeward side, the responses are promoted. In contrast, when the mass center is placed at the windward side, it leads to better instability of the bridge. This conclusion agrees with that from the work by Gu et al. (2000).

In the present case of study, the mechanism of the occurrence of flutter can be explored by examining the variations of the net damping ratios. Figure 10a shows that the net damping in the cross-wind direction is always larger than the material damping. (The same evidence can also be detected in Figure 9a, which shows all negative $H^*$ values in the entire range of $U_r$.) By investigating the result in Figure 10b, on the other hand, it appears that the net damping in the torsional direction drops to zero at the critical speed. Accordingly, one can then conclude that although the fluctuating responses diverge in both directions (Figs 3-8) at the critical flutter speeds, the instability of the deck is in reality initiated by torsional flutter. It is also noted that the net torsional damping becomes identical to that of the material damping as $U_r$ is about 4.70 (Fig. 10b), corresponding to the instant that $A_2^*$ changes its sign (Fig. 9b).

7 CONCLUSIONS

The proposed numerical model has been proved to be adequate in predicting the wind effects as well as the dynamic responses of a suspension bridge with a hexagonal cross-section. The effect of the attack angle, mass eccentricity and frequency ratio are investigated in the study. Finally, besides the predictions of the deck responses, the interaction mechanism of the vibrating bridge deck is discussed extensively based on the numerical results.

8 ACKNOWLEDGEMENT

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9 REFERENCES


Table 1  Related properties of experiments

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<tr>
<th>Frequency Ratio ($f_t/f_v$)</th>
<th>Mass (M) (kg/m)</th>
<th>Moment of Inertia (I) (kg·m²/m)</th>
<th>Fundamental Frequency (Hz)</th>
<th>Damping ratio (%)</th>
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<td></td>
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<td>Vertical ($f_v$)</td>
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<td>$9.8 \times 10^{-3}$</td>
<td>8.58</td>
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Table 2  Calculated normalized critical velocities

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<th>$f_t/f_v$</th>
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<tr>
<td>$\beta/e/B$</td>
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<td>0.1</td>
<td>8.19</td>
<td>7.51</td>
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<td></td>
<td>(-15.6 %)*</td>
<td>(-13.6 %)</td>
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<tr>
<td>0</td>
<td>7.71</td>
<td>7.10</td>
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<td></td>
<td>(-14.5 %)</td>
<td>(-12.9 %)</td>
</tr>
<tr>
<td>−0.1</td>
<td>7.35</td>
<td>6.79</td>
</tr>
<tr>
<td></td>
<td>(-13.8 %)</td>
<td>(-11.89 %)</td>
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* Values in the brackets indicate the percentages of reduction of normalized critical velocity.

Figure 1  Schematic of the problem

Figure 2  Experimental setup
Figure 3  Root-mean-square deflections at various reduced velocities ($f_t/f_v=1.21$, $e/B=0$)

Figure 4  Root-mean-square deflections at various reduced velocities ($f_t/f_v=1.21$, $e/B=0.1$)

Figure 5  Root-mean-square deflections at various reduced velocities ($f_t/f_v=1.21$, $e/B=-0.1$)
Figure 6  Root-mean-square deflections at various reduced velocities ($f_r/f_v=1.01, e/B=0$)

Figure 7  Root-mean-square deflections at various reduced velocities ($f_r/f_v=1.01, e/B=0.1$)

Figure 8  Root-mean-square deflections at various reduced velocities ($f_r/f_v=1.01, e/B=-0.1$)
Figure 9 Calculated flutter derivatives at various reduced velocities ($f_f/f_v=1.01$, $e/B=-0.1$, $\beta=8^\circ$)

Figure 10 Calculated net damping ratios at various reduced velocities ($f_f/f_v=1.01$, $\beta=8^\circ$)